3.1 Exercise Set

1. Determine the quadrant in which the terminal side of the angle lies.

a) 70° I

b) $-70°$ IV

c) 191° III

d) $-191°$ II

e) 306° IV

f) $-306°$ I

g) 162° II

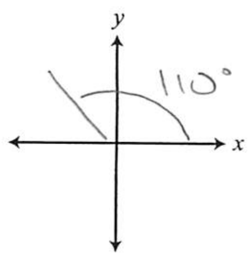
h) $-162°$ III

i) 400° I

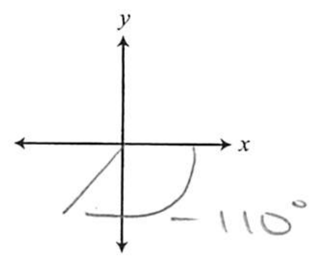
j) $-400°$ IV

2. Sketch the angles in standard position.

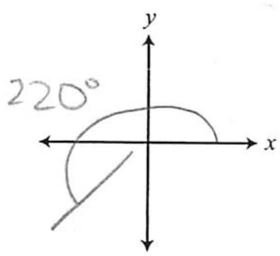
a) 110°



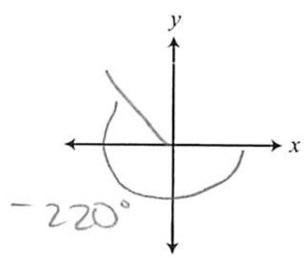
b) $-110°$



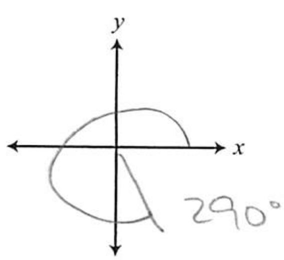
c) 220°



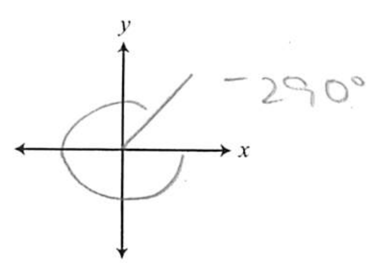
d) $-220°$



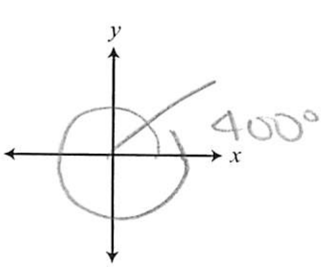
e) 290°



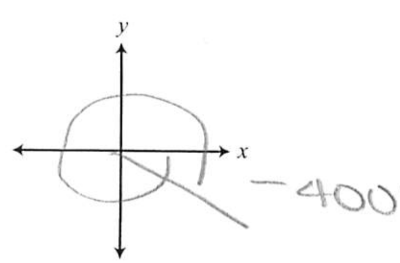
f) $-290°$



g) 400°



h) $-400°$



3. Determine one positive and one negative coterminal angle for the given angle.

a) 41° +360 −360 401°, −319° b) −41° ______

c) 123° 483°, −237° d) −123° ______

e) 90° ±360 450°, −270° f) 180° ______

g) 243° ±360 603°, −117° h) −243° ______

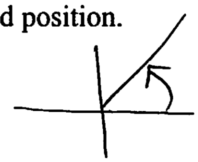
i) 297° ±360 657°, −63° j) −297° ______

k) 370° +360 −360−360 730°, −350° l) −370° ______

m) 430° +360 −360−360 790°, −290° n) −430° ______

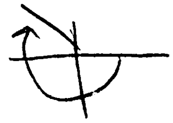
4. Give the degree measure of each angle in standard position.

a) One-sixth of a revolution counter-clockwise. 60°



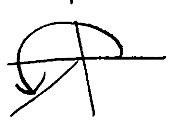
$\frac{1}{6}(360) = 60°$

b) Three-fifths of a revolution clockwise. −216°



$\frac{3}{5}(360) = 216°$

c) Five-eighths of a revolution counter-clockwise. 225°



$\frac{5}{8}(360) = 225°$

d) One-fourth of a revolution clockwise. −90°

$\frac{1}{4}(360) = 90°$

e) One and one-half revolutions counter-clockwise. 540°

$\frac{3}{2}(360) = 540°$

f) Two and one-quarter revolutions clockwise. −810°

$\frac{9}{4}(360) = 810°$

g) One-tenth of a revolution counter-clockwise. 36°

$\frac{1}{10}(360) = 36°$

h) One-eighth of a revolution clockwise −45°

$\frac{1}{8}(360) = 45°$

5. Find the angle of smallest positive measure that is coterminal with the given angle.

a) $-30°$ $+360°$ $\underline{330°}$ **b)** $-96°$ ______

c) $-197°$ $+360°$ $\underline{163°}$ **d)** $-314°$ ______

e) $-127°$ $+360°$ $\underline{233°}$ **f)** $405°$ ______

g) $502°$ $-360°$ $\underline{142°}$ **h)** $437°$ ______

i) $615°$ $-360°$ $\underline{255°}$ **j)** $-475°$ ______

6. Give an expression that generates all angles coterminal with the given angle. Let n represent any integer.

a) $30° + 360°n$ ______ **b)** $-100° + 360°n$ ______

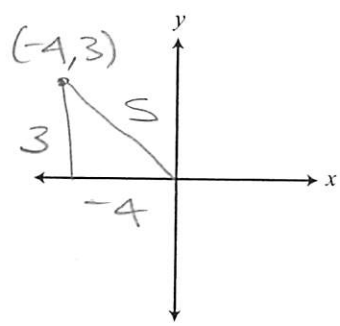
c) $130° + 360°n$. ______ **d)** $-215° + 360°n$ ______

7. Sketch an angle θ in standard position such that θ has the smallest positive measure, and the given point is on the terminal side of θ and determine the radius r.

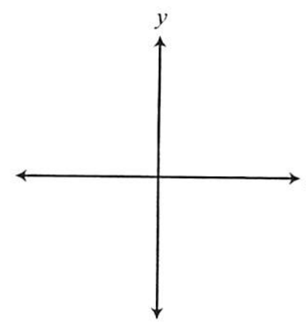
a) $(-4, 3)$

$r = \sqrt{3^2 + (-4)^2}$

$r = 5$



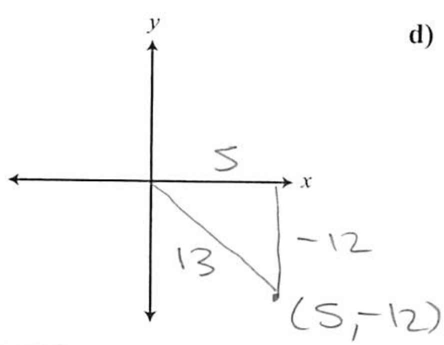
b) $(3, 4)$



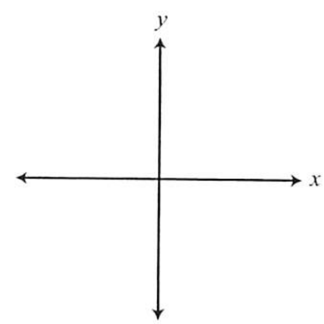
c) $(5, -12)$

$r = \sqrt{5^2 + (-12)^2}$

$r = 13$



d) $(-12, -5)$



II S | A I

III T | C IV

8. Suppose that the point (x, y) is in the indicated quadrant. Decide whether the given ratio is positive or negative.

a) II, $\frac{x}{r}$ cos + − b) II, $\frac{y}{r}$ sin + −

c) III, $\frac{x}{r}$ cos + − d) III, $\frac{y}{r}$ sin + −

e) IV, $\frac{x}{r}$ cos + − f) IV, $\frac{y}{r}$ sin + −

g) I, $\frac{y}{x}$ tan + − h) II, $\frac{y}{x}$ tan + −

i) III, $\frac{y}{x}$ tan + − j) IV, $\frac{y}{x}$ tan + −

9. Find the reference angle for the following.

a) 32° 32° b) −32° ______

c) 113° 180−113 67° d) −113° ______

e) 218° 218−180 38° f) −218° ______

g) 304° 360−304 56° h) −304° ______

i) 832° 180−112 68° j) −1213° ______

− 360
− 360
112

10. Find all angles, $0° \le \theta < 360°$, that have reference angles of:

a) 37° 37°, 143°, 217°, 323° b) 71°

c) 0° 0°, 180° d) 90°

11. If quadrants II and III have the same reference angles and one of the standard position angles is 214°, what is the other smallest positive standard position angle in quadrant II? 214 − 180 = 34

214°

180 − 34 = 146°

12. If quadrants III and IV have the same reference angle and the standard position angle in quadrant IV is 333°, what is the smallest positive standard position angle in quadrant III?

II S | A I

III T | C IV

3.2 Exercise Set

1. Identify the quadrant(s) for the angles satisfying the following conditions.

a) $\cos\rho < 0$ II, III

b) $\tan\theta > 0$ ______

c) $\sin\alpha < 0$ III, IV

d) $\sin\alpha > 0,\ \tan\alpha < 0$ ______

e) $\cos\beta < 0,\ \tan\beta > 0$ III

II III   I III

f) $\sin\phi < 0,\ \cos\phi > 0$ ______

g) $\sin\delta > 0,\ \tan\delta > 0$ I

I II   I III

h) $\cos\gamma < 0,\ \tan\gamma < 0$ ______

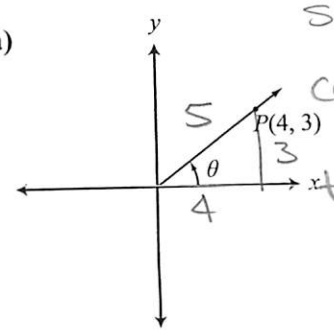
i) $\sin\tau < 0,\ \tan\tau < 0$ IV

III IV   II IV

j) $\sin\sigma < 0,\ \cos\sigma < 0$ ______

2. A point P on the terminal side of θ is shown. Evaluate the three trigonometric functions of θ.

a)

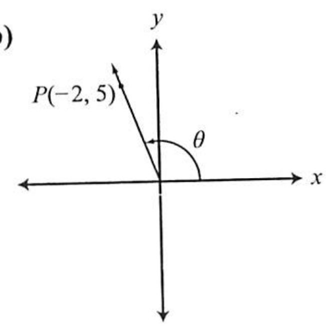


$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$

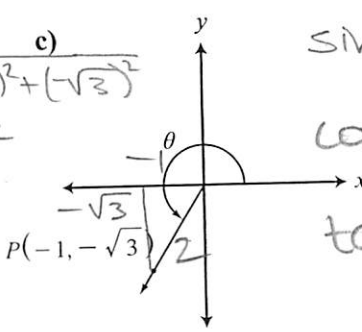
b)



c)

$r = \sqrt{(-1)^2 + (-\sqrt{3})^2}$

$r = 2$

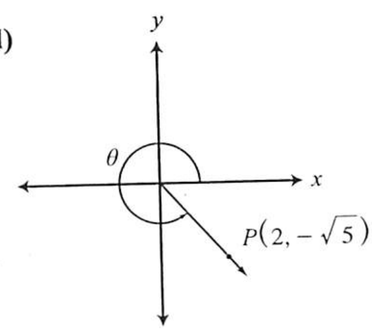


$\sin\theta = -\frac{\sqrt{3}}{2}$

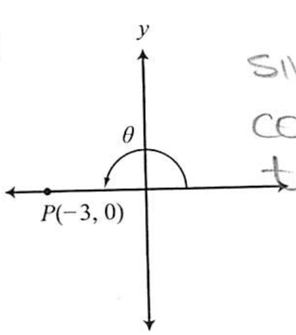
$\cos\theta = -\frac{1}{2}$

$\tan\theta = \frac{-\sqrt{3}}{-1} = \sqrt{3}$

d)



e)

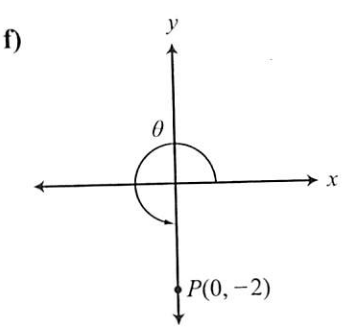


$\sin\theta = 0$

$\cos\theta = -1$

$\tan\theta = 0$

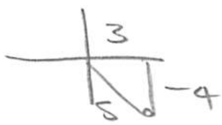
f)



3. If θ is in standard position and the given point is on the terminal side of θ, find the values of $\sin\theta$, $\cos\theta$, and $\tan\theta$.

a) $(3,-4)$

$\sin\theta = \frac{-4}{5}$



$\cos\theta = \frac{3}{5}$

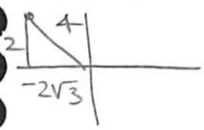
$\tan\theta = -\frac{4}{3}$

b) $(-12,5)$

c) $(-7,-24)$

d) $(8,15)$

e) $(-2\sqrt{3},2)$ $r=\sqrt{4+4(3)}=\sqrt{16}=4$



$\sin\theta = \frac{2}{4} = \frac{1}{2}$

$\cos\theta = \frac{-2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$

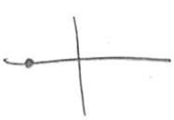
$\tan\theta = \frac{2}{-2\sqrt{3}} = -\frac{1}{\sqrt{3}}$

f) $(\sqrt{2},\sqrt{7})$

g) $(-3,-3)$

h) $(0,4)$

i) $(-2,0)$



$\sin\theta = 0$

$\cos\theta = -1$

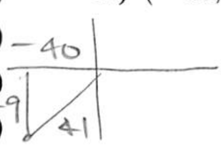
$\tan\theta = 0$

j) $(-\sqrt{5},2)$

k) $(-5,5)$

l) $(-2\sqrt{2},1)$

m) $(-40,-9)$ $r=\sqrt{1600+81}=\sqrt{1681}=41$



$\sin\theta = \frac{-9}{41}$

$\cos\theta = \frac{-40}{41}$

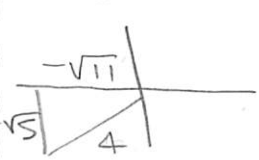
$\tan\theta = \frac{-9}{-40} = \frac{9}{40}$

n) $(9,-40)$

o) $(2\sqrt{2},8)$

p) $(-3,\sqrt{3})$

q) $(-\sqrt{11},-\sqrt{5})$ $r=\sqrt{11+5}=\sqrt{16}=4$



$\sin\theta = -\frac{\sqrt{5}}{4}$

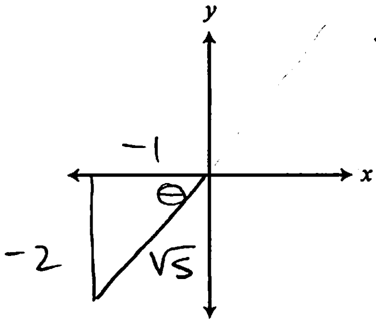
$\cos\theta = -\frac{\sqrt{11}}{4}$

$\tan\theta = \frac{-\sqrt{5}}{-\sqrt{11}} = \frac{\sqrt{5}}{\sqrt{11}}$

r) $(\sqrt{17},2\sqrt{2})$

4. An equation with a restriction on x is given. This is an equation of the terminal side of an angle in standard position. Draw the smallest positive angle θ, and find the values of $\sin\theta$, $\cos\theta$, and $\tan\theta$.

a) $y = 2x,\ x \le 0$ $\quad r = \sqrt{4+1} = \sqrt{5}$

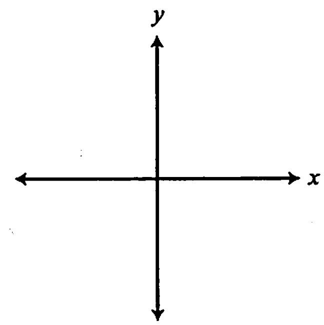


$\sin\theta = \dfrac{-2}{\sqrt{5}}$

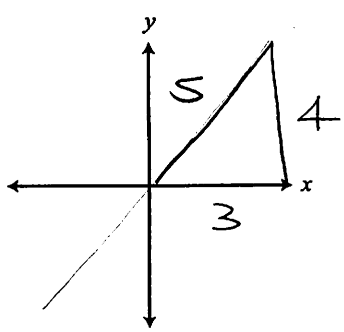
$\cos\theta = \dfrac{-1}{\sqrt{5}}$

$\tan\theta = \dfrac{-2}{-1} = 2$

b) $y = -\frac{2}{5}x,\ x \ge 0$



c) $y = \frac{4}{3}x,\ x \ge 0$ $\quad r = \sqrt{16+9} = 5$

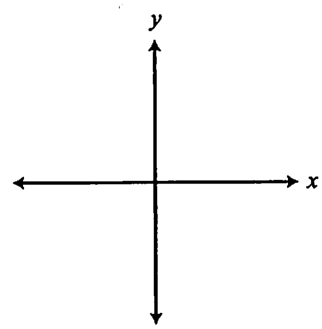


$\sin\theta = \dfrac{4}{5}$

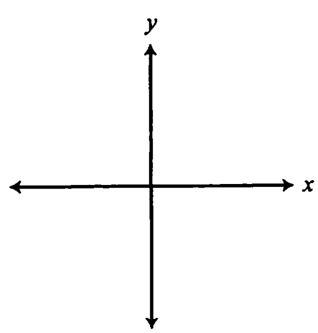
$\cos\theta = \dfrac{3}{5}$

$\tan = \dfrac{4}{3}$

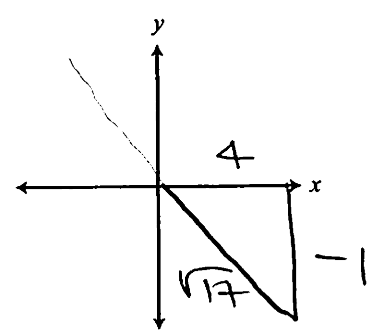
d) $y = -\frac{3}{2}x,\ x \le 0$



e) $y = \frac{5}{2}x,\ x \le 0$



f) $y = -\frac{1}{4}x,\ x \ge 0$ $\quad r = \sqrt{16+1} = \sqrt{17}$

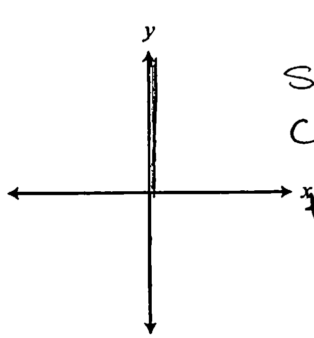


$\sin\theta = \dfrac{-1}{\sqrt{17}}$

$\cos\theta = \dfrac{4}{\sqrt{17}}$

$\tan\theta = \dfrac{-1}{4}$

g) $x = 0,\ y \ge 0$

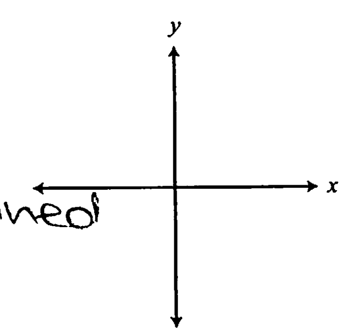


$\sin\theta = 1$

$\cos\theta = 0$

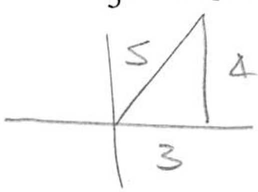
$\tan\theta =$ undefined

h) $y = 0,\ x \le 0$



5. The value of one of the trigonometric functions is given, along with some additional information. Use this information to find the other two trigonometric functions of θ.

a) $\sin\theta = \frac{4}{5}$, θ in quadrant I



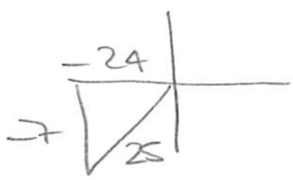
$\cos\theta = \frac{3}{5}$

$\tan\theta = \frac{4}{3}$

b) $\cos\theta = -\frac{12}{13}$, θ in quadrant II

c) $\tan\theta = \frac{7}{24}$, θ in quadrant III $r = \sqrt{24^2+7^2}$

$= 25$



$\sin\theta = -\frac{7}{25}$

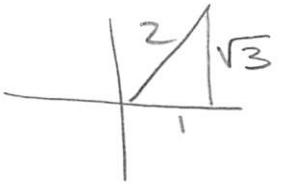
$\cos\theta = -\frac{24}{25}$

d) $\sin\theta = -\frac{\sqrt{3}}{2}$, θ in quadrant IV

e) $\cos\theta = \frac{1}{2}$, $\tan\theta > 0$

I IV   I III

$y = \sqrt{4-1}$



$\sin\theta = \frac{\sqrt{3}}{2}$

$\tan\theta = \frac{\sqrt{3}}{1} = \sqrt{3}$

f) $\tan\theta = \frac{2}{\sqrt{5}}$, $\sin\theta > 0$

g) $\sin\theta = -\frac{3}{\sqrt{10}}$, $\cos\theta < 0$

III IV   II III

$x = \sqrt{10-9} = 1$

$\cos\theta = -\frac{1}{\sqrt{10}}$

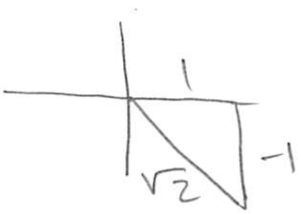
$\tan\theta = \frac{-3}{-1} = 3$

h) $\cos\theta = \frac{3}{\sqrt{13}}$, $\tan\theta < 0$

i) $\tan\theta = -1$, $\sin\theta < 0$

II IV   III IV

$r = \sqrt{1+1} = \sqrt{2}$



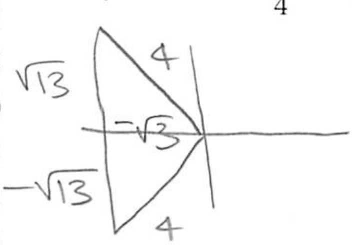
$\sin\theta = -\frac{1}{\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}}$

j) $\sin\theta = \frac{\sqrt{15}}{4}$

k) $\cos\theta = -\frac{\sqrt{3}}{4}$

$y = \sqrt{16-3} = \sqrt{13}$



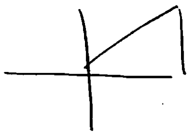
$\sin\theta = \pm\frac{\sqrt{13}}{4}$

$\tan\theta = \pm\frac{\sqrt{13}}{\sqrt{3}} = \pm\frac{\sqrt{39}}{3}$

l) $\tan\theta = -\sqrt{3}$

6. The value of one of the trigonometric functions is given along with some additional information. Use the trigonometric ratios to find the other two trigonometric functions of θ Round each answer to three decimal places.

a) $\sin\theta = 0.642$, θ in quadrant I

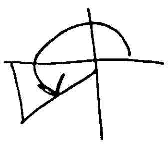


$\theta = 39.941°$
$\cos\theta = 0.767$
$\tan\theta = 0.837$

b) $\cos\theta = 0.537$, θ in quadrant IV

c) $\tan\theta = 2$, θ in quadrant III

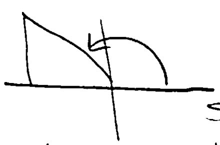
$\theta_R = 63.435°$



$\theta = 243.345°$
$\sin\theta = -0.894$
$\cos\theta = -0.449$

d) $\sin\theta = 0.237$, θ in quadrant II

II III　　　I II
e) $\cos\theta = -0.378$, $\sin\theta > 0$

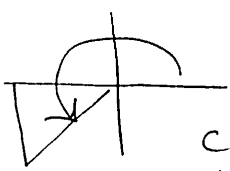


$\theta = 112.210°$
$\sin\theta = 0.926$
$\tan\theta = -2.449$

f) $\tan\theta = -1.413$, $\cos\theta > 0$

III IV　　　I III
g) $\sin\theta = -0.753$, $\tan\theta > 0$

$\theta_R = 48.851°$

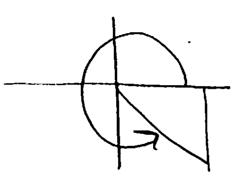


$\theta = 228.851°$
$\cos\theta = -0.658$
$\tan\theta = 1.144$

h) $\cos\theta = -0.492$, $\sin\theta > 0$

II IV　　　III IV
i) $\tan\theta = -0.866$, $\sin\theta < 0$

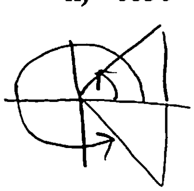
$\theta_R = 40.893°$



$\theta = 319.107°$
$\sin\theta = -0.655$
$\cos\theta = 0.756$

j) $\sin\theta = -0.351$

k) $\cos\theta = 0.811$　$\theta_R = 35.806$



$\theta_1 = 35.806°$
$\theta_2 = 324.194°$
$\sin\theta = \pm 0.585$
$\tan\theta = \pm 0.721$

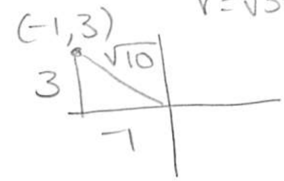
l) $\tan\theta = 0.463$

7. The terminal side of angle θ passes through the intersection point of the given lines. Find the three trigonometric functions of θ.

$r=\sqrt{3^2+1^2}$

a) $2x + y = 1$
$+3x + y = 6$ (written: $-3x - y = -6$)

$5x = -5$
$x = -1$



$2(-1)+y=1$
$y=1+2$
$y=3$

$\sin\theta = \dfrac{3}{\sqrt{10}}$

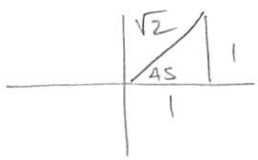
$\cos\theta = \dfrac{-1}{\sqrt{10}}$

$\tan\theta = \dfrac{3}{-1} = -3$

b) $3x + y = 10$
$x - 4y = 12$

8. Find the exact value of the three trigonometric functions of θ if θ is in standard position and the terminal side of θ is in the specified quadrant and satisfies the given condition.

a) I; bisects the quadrant

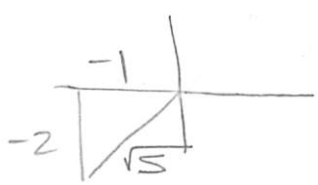


$\sin\theta = 1/\sqrt{2}$ $\quad\tan\theta = 1$
$\cos\theta = 1/\sqrt{2}$

b) II; parallel to the line $3x + 2y = 2$

c) III; parallel to the line through $A(1, 4)$ and $B(-3, -4)$

$m_{AB} = \dfrac{-4-4}{-3-1} = \dfrac{-8}{-4} = 2$



$r=\sqrt{2^2+1^2}$

$\sin\theta = \dfrac{-2}{\sqrt{5}}$

$\cos\theta = \dfrac{-1}{\sqrt{5}}$

$\tan\theta = \dfrac{-2}{-1} = 2$

d) IV; parallel to the line through $A(-2, 4)$ and $B(5, -2)$

9. Using different values of θ, evaluate $\sin\theta$ and $\sin(-\theta)$. How does the value of $\sin(-\theta)$ compare to the value of $\sin\theta$?

$\sin 90 = 1$ $\quad \sin(-90) = -1$
$\sin 30 = 0.5$ $\quad \sin(-30) = -0.5$
$\sin(-\theta) = -\sin\theta$

10. Using different values of θ, evaluate $\cos\theta$ and $\cos(-\theta)$. How does the value of $\cos(-\theta)$ compare to the value of $\cos\theta$?

3.3 Exercise Set

1. Find the reference angle α for each angle θ.

a) 300° 60° **b)** 135° ____

c) 240° 60° **d)** 120° ____

e) 330° 30° **f)** 150° ____

g) 111° 69° **h)** 200° ____

i) 280° 80° **j)** 180° ____

k) 73° 73° **l)** 91° ____

m) 179° 1° **n)** 270° ____

2. Find the angle θ for each reference angle α, in the given quadrant.

a) 30°, II 150° **b)** 45°, III ____

c) 60°, IV 300° **d)** 30°, III ____

e) 45°, II 135° **f)** 60°, II ____

g) 30°, IV 330° **h)** 45°, IV ____

i) 60°, III 240° **j)** 37°, II ____

k) 37°, III 217° **l)** 37°, IV ____

3. Evaluate exactly without using a calculator or table. Leave answers involving radicals in exact radical form.

a) $\sin 0°$ $\underline{0}$ b) $\cos 0°$ ______

c) $\tan 0°$ $\underline{0}$ d) $\sin 270°$ ______

e) $\cos 270°$ $\underline{0}$ f) $\tan 270°$ ______

g) $\sin 120°$ $\underline{\sqrt{3}/2}$ h) $\cos 120°$ ______

i) $\tan 120°$ $\underline{-\sqrt{3}}$ j) $\sin 315°$ ______

k) $\cos 315°$ $\underline{1/\sqrt{2}}$ l) $\tan 315°$ ______

m) $\sin 210°$ $\underline{-1/2}$ n) $\cos 210°$ ______

o) $\tan 210°$ $\underline{1/\sqrt{3}}$ p) $\tan 180°$ ______

4. Find all θ, $0° \le \theta < 360°$, which satisfy each equation.

a) $\sin\theta = \frac{\sqrt{3}}{2}$ $\theta = 60°, 120°$ b) $\cos\theta = \frac{\sqrt{3}}{2}$

c) $\tan\theta = -\frac{1}{\sqrt{3}}$ $\theta = 150°, 330°$ d) $\sin\theta = -\frac{1}{\sqrt{2}}$

e) $\cos\theta = -\frac{1}{\sqrt{2}}$ $\theta = 135°, 225°$ f) $\tan\theta = -1$

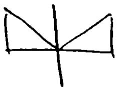
g) $\sin\theta = 0$ $\theta = 0°, 180°$ h) $\cos\theta = 0$

i) $\tan\theta = 0$ $\theta = 0°, 180°$ j) $\sin\theta = -1$

k) $\cos\theta = -\frac{1}{2}$ $\theta = 120°, 240°$ l) $\tan\theta = \sqrt{3}$

5. Find, to one decimal place, all θ, $0° \le \theta < 360°$, which satisfy each equation.

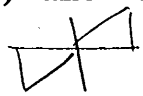
a) $\sin\theta = 0.253$



$\theta = 14.7°, 165.3°$

b) $\cos\theta = 0.425$

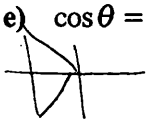
c) $\tan\theta = 2$



$\theta = 63.4°, 243.4°$

d) $\sin\theta = -0.625$

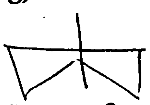
e) $\cos\theta = -0.738$



$\theta = 137.6°, 222.4°$

f) $\tan\theta = -0.543$

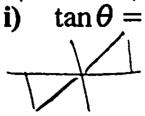
g) $\sin\theta = -0.123$



$\theta = 187.1°, 352.9°$

h) $\cos\theta = -0.123$

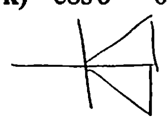
i) $\tan\theta = 0.001$



$\theta = 0.1°, 180.1°$

j) $\sin\theta = 0.427$

k) $\cos\theta = 0.222$

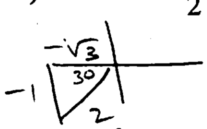


$\theta = 77.2°, 282.8°$

l) $\tan\theta = -5000$

6. Find the smallest positive angle θ, $0° \le \theta < 360°$, which satisfies each equation.

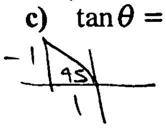
a) $\sin\theta = -\frac{1}{2}$



$\theta = 210°$

b) $\cos\theta = -\frac{1}{2}$

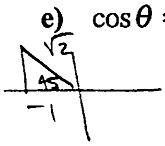
c) $\tan\theta = -1$



$\theta = 135°$

d) $\sin\theta = -\frac{1}{\sqrt{2}}$

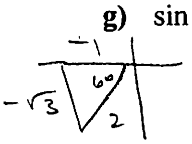
e) $\cos\theta = -\frac{1}{\sqrt{2}}$



$\theta = 135°$

f) $\tan\theta = -\sqrt{3}$

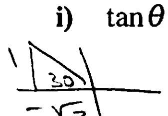
g) $\sin\theta = -\frac{\sqrt{3}}{2}$



$\theta = 240°$

h) $\cos\theta = -\frac{\sqrt{3}}{2}$

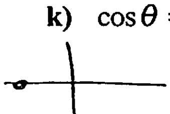
i) $\tan\theta = -\frac{1}{\sqrt{3}}$



$\theta = 150°$

j) $\sin\theta = -1$

k) $\cos\theta = -1$



$\theta = 180°$

l) $\tan\theta =$ undefined

7. Find the area of an equilateral triangle with sides of 10 cm in length.

$(\sqrt{3}\times 5)$ $10\ (2\times 5)$ 60 5 (1×5)

$\text{area} = \dfrac{bh}{2}$

$\text{area} = \dfrac{10\cdot 5\sqrt{3}}{2}$

$\text{area} = 25\sqrt{3}\,\text{cm}^2$

8. Find the area of a triangle with sides of length 8 cm and 10 cm, and an angle between the two given sides of 74°.

9. Find the area of a triangle with sides of length 5 cm and 10 cm, and an angle between the two given sides of 110°.

h 5 110 70 10

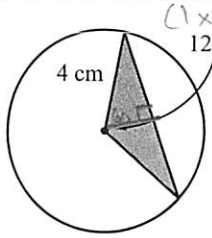
$\sin 70 = \dfrac{h}{5}$

$h = 5\sin 70$

$A = \dfrac{10\times 5\sin 70}{2} = 23.49\,\text{cm}^2$

10. A triangle has an area of 15 mm², and two sides of the triangle are 6 mm and 8 mm. Find the angle between the two given sides of the triangle.

11. Find the area of the shaded region.

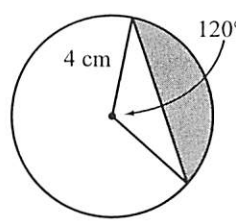


(1×2) $60°$ $4\ (2\times 2)$ $(\sqrt{3}\times 2)$

2 $4\sqrt{3}$

$A = \dfrac{bh}{2} = \dfrac{4\sqrt{3}\cdot \cancel{2}}{\cancel{2}} = 4\sqrt{3}\,\text{cm}^2$

12. Find the area of the shaded region in terms of π.



13. A regular hexagon is inscribed in a circle of radius 10 cm. Find the area of the hexagon.

10 10 10 $10\ (2\times 5)$ 5 $(\sqrt{3}\times 5)$ (1×5)

$\text{area}_\triangle = \dfrac{10\cdot 5\sqrt{3}}{2} = 25\sqrt{3}$

$\times 6 = 150\sqrt{3}\,\text{cm}^2$

14. A regular nonagon (nine sides) is inscribed in a circle of radius 10 cm. Find the area of the nonagon.

3.5 Exercise Set

1. Explain why no triangle is possible with the given information.

largest ∠

a) $A = 38°$ $B = 69°$ $C = 73°$
$a = 12$ $b = 14$ $c = 13$

largest side should be opposite

b) $A = 42°$ $B = 65°$ $C = 70°$
$a = 7$ $b = 11$ $c = 12$

c) $A = 39°$ $B = 46°$ $C = 95°$
$a = 5$ $b = 6$ $c = 12$

sum must be greater than 12

d) $A = 120°$ $B = 20°$ $C = 40°$
$a = 12$ $b = 6$ $c = 12$

2. Find the sine angle equivalent to the following, $0° \leq \theta \leq 180°$.

a) $\sin 10°$ 170°

b) $\sin 30°$ ______

c) $\sin 42°$ 138°

d) $\sin 71°$ ______

e) $\sin 121°$ 59°

f) $\sin 137°$ ______

3. Determine if the set of data leads to 0, 1, or 2 triangles. A drawing may be helpful.

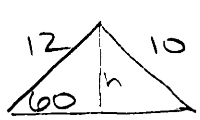
a) $\angle A = 60°,\ a = 6\sqrt{3},\ b = 12$ 1

$h = 12\sin 60 = 10.4$
$6\sqrt{3} = 10.4$
$a = h$

b) $\angle A = 60°,\ a = 11,\ b = 12$ 2

$\frac{11}{12}$ $\sin 60$
$0.917 > 0.866$
$\sin A < \frac{a}{b} < 1$

c) $\angle A = 60°,\ a = 10,\ b = 12$ 0



$h = 12\sin 60 = 10.4$
$a = 10$
$h > a$

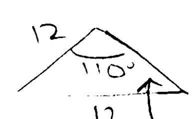
d) $\angle A = 60°,\ a = 12,\ b = 12$ 1

$\frac{12}{12} = 1$

e) $\angle A = 110°,\ a = 16,\ b = 12$ 1

$a > b$

f) $\angle A = 110°,\ a = 12,\ b = 12$ 0



can't equal 110°

4. Given $\angle A$ and side b, determine the lengths for side a that allow 0, 1, or 2 triangles to be formed.

a) $\angle A = 30°,\ b = 12$

$a < 12\sin 30$
$a < 6 \rightarrow$ zero

$a = 12\sin 30$
$a = 6 \rightarrow$ one
or $a \geq 12$

$12\sin 30 < a < 12$
$6 < a < 12$
$\rightarrow$ two

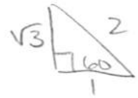
b) $\angle A = 45°,\ b = 4\sqrt{2}$

$a < 4\sqrt{2}\sin 45$
$a < 4\sqrt{2}(1/\sqrt{2})$
$a < 4 \rightarrow$ zero

$a = 4 \rightarrow$ one
or $a \geq 4\sqrt{2}$

$4 < a < 4\sqrt{2}$
$\rightarrow$ two.

c) $\angle A = 60°,\ b = 6\sqrt{3}$



$a < 6\sqrt{3}\sin 60$
$a < 6\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$ $a < 9. \rightarrow$ zero

$a = 6\sqrt{3}\sin 60$
$a = 9$
$\rightarrow$ one.
or $a \geq 6\sqrt{3}$

$6\sqrt{3}\sin 60 < a < 6\sqrt{3}$.
$9 < a < 6\sqrt{3}$
$\rightarrow$ two.

d) $\angle A = 120°,\ b = 12$

obtuse

$a > 12 \rightarrow 1\ \triangle$'s
$a \leq 12 \rightarrow 0\ \triangle$'s

5. Solve for the unknown angle, if possible, then determine if a second angle, $0° < \theta < 180°$, exists that will satisfy the proportion but not the triangle.

a) $\dfrac{\sin A}{10} = \dfrac{\sin 40°}{30}$

$\sin^{-1}\left(\frac{\sin 40}{30} \cdot 10\right) = 12.4°$

$180 - 12.4 = 167.6°$

b) $\dfrac{\sin A}{200} = \dfrac{\sin 20°}{50}$

$\frac{\sin 20}{50} \cdot 200 > 1$

$\varnothing$

c) $\dfrac{\sin A}{10} = \dfrac{\sin 30°}{5}$

$\sin^{-1}\left(\frac{\sin 30}{5} \cdot 10\right) = 90°$

d) $\dfrac{\sin A}{40} = \dfrac{\sin 57°}{53}$

$\sin^{-1}\left(\frac{\sin 57}{53} \cdot 40\right) = 39.3°$

or $140.7°$

e) $\dfrac{\sin A}{3} = \dfrac{\sin 125°}{5}$

$\sin^{-1}\left(\frac{\sin 125}{5} \cdot 3\right) = 29.4°$

or $150.6°$

f) $\dfrac{\sin A}{7.3} = \dfrac{\sin 12°}{1.3}$

$\sin^{-1}\Bigg(\underbrace{\frac{\sin 12 \cdot 7.3}{1.3}}_{>1}\Bigg) = \varnothing$

 ☆ careful with ASS !

6. Solve using the Law of Sines. ~~If two triangles~~ exist, solve both completely. A drawing is very helpful.

$\boxed{\angle B = 15°}$

a) $\angle A = 140°$, $\angle C = 25°$, $a = 20$ AAS

$\frac{\sin 140}{20} = \frac{\sin 25}{c}$ $\quad \frac{\sin 140}{20} = \frac{\sin 15}{b}$

$c = \frac{20 \sin 25}{\sin 140} = \boxed{13.1}$ $\quad \boxed{b = 8.1}$

b) $\angle B = 38°$, $b = 8$, $a = 6$ ASS

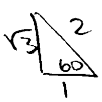
c) ~~$\angle C = 27°$, $\angle B$~~ $= 46°$, $a = 120$ AAS

$\boxed{\angle A = 107°}$

$\frac{\sin 27}{c} = \frac{\sin 107}{120}$ $\quad c = \boxed{57.0}$

$\frac{\sin 46}{b} = \frac{\sin 107}{120}$ $\quad b = \boxed{90.3}$

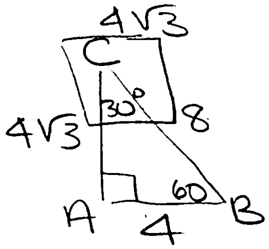
d) $\angle A = 110°$, $a = 24$, $b = 25$ ASS



e) $\angle B = 60°$, $b = 4\sqrt{3}$, $a = 8$ ASS

$\frac{\sin 60}{4\sqrt{3}} = \frac{\sin A}{8}$ $\quad \sin A = 1$

$\boxed{A = 90°}$



$c = \sqrt{8^2 - (4\sqrt{3})^2}$

$\boxed{c = 4}$

f) $\angle C = 41°$, $c = 9$, $a = 9$ ASS

g) $\angle A = 74°$, $a = 7$, $b = 8.1$ ASS

$\frac{\sin 74}{7} = \frac{\sin B}{8.1}$

$\frac{8.1 \cdot \sin 74}{7} > 1$

$\therefore \varnothing$

h) $\angle A = 58°$, $\angle B = 48°$, $b = 30.5$ AAS.

i) $\angle A = 43°$, $\angle B = 38°$, $c = 17.2$ AAS

$\boxed{\angle C = 99°}$

$\frac{\sin 43}{a} = \frac{\sin 99}{17.2}$ $\quad a = \boxed{11.9}$

$\frac{\sin 38}{b} = \frac{\sin 99}{17.2}$ $\quad b = \boxed{10.7}$

j) $\angle A = 33°$, $a = 27.2$, $b = 12.4$ ASS.

6. k) $\angle A = 30°,\ a = 8,\ b = 10$ ASS.

$\frac{\sin 30}{8} = \frac{\sin B}{10} = \frac{\sin 111.3}{c}$ or $\frac{\sin 8.7}{c}$

$B = 38.7°$ $\boxed{c = 14.9 \text{ or } 2.4}$

$10\sin 30 < 8 < 10.$
5

$\boxed{\begin{matrix}\therefore \angle B = 38.7°,\ 141.3° \\ \angle C = 111.3°,\ 8.7°\end{matrix}}$

l) $\angle A = 58°,\ a = 9,\ b = 10$

m) $\angle A = 10°,\ \angle B = 60°,\ a = 4.5$ AAS.

$\angle C = \boxed{110°}$

$\frac{\sin 10}{4.5} = \frac{\sin 60}{b}$ $b = \boxed{22.4}$

$\frac{\sin 10}{4.5} = \frac{\sin 110}{c}$ $c = \boxed{24.4}$

n) $\angle B = 10°,\ \angle C = 135°,\ c = 60$

o) $\angle C = 52°,\ c = 8.5,\ b = 12.4$ ASS

$12.4\sin 52 \cancel{<} 8.5 < 12.4$

$\frac{\sin 52}{8.5} = \frac{\sin B}{12.4}$

$\frac{12.4 \cdot \sin 52}{8.5} > 1$ $\varnothing$.

p) $\angle B = 27,\ b = 2,\ c = 5$

q) $\angle B = 27°,\ b = 4,\ c = 5$ ASS

$5\sin 27 < 4 < 5$
2.3

$\therefore$ 2 $\triangle$'s.

$\frac{\sin 27}{4} = \frac{\sin C}{5}$ $\angle C = \boxed{34.6° \text{ or } 145.4°}$

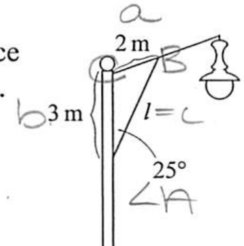
$\angle A = \boxed{118.4° \text{ or } 7.6°}$

$\frac{\sin 27}{4} = \frac{\sin 118.4}{a}$ $a = \boxed{7.8}$ or $\boxed{1.2}$

$\frac{\sin 27}{4} = \frac{\sin 7.6}{a}$

r) $\angle B = 40°,\ b = 55,\ c = 80$

7. Find the length, l, of the brace required to support the lamp.



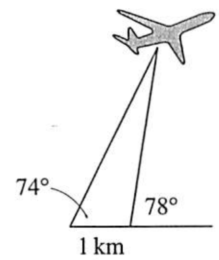
$3\sin 25 < 2 < 3$
1.3
$\therefore$ 2 △'s.

$\frac{\sin 25}{2} = \frac{\sin B}{3}$ $\angle B = 39.3°$ or $140.7°$
$\angle C = 115.7°$ or $14.3°$.

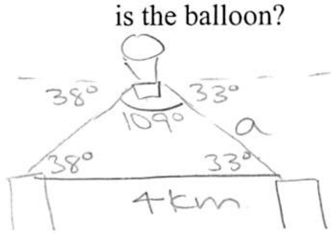
$\frac{\sin 115.7}{c} = \frac{\sin 25}{2}$ or $\frac{\sin 14.3}{c} = \frac{\sin 25}{2}$.

$c = 4.3\text{m}$ or $c = 1.2\text{m}$.
if you assume ∠C is obtuse

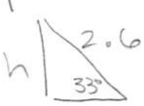
8. A plane is sighted by two observers 1 km apart at angles 74° and 78°. How high is the plane?



9. A hot air balloon is flying directly between two cities that are 4 km apart. The balloonist finds that the angle of depression to one city is 38° and 33° to the other city. How high above the ground is the balloon?



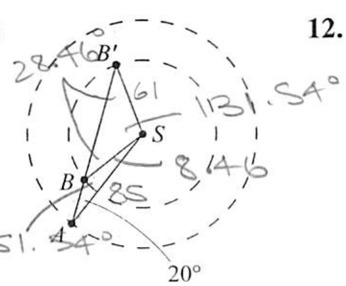
$\frac{\sin 109}{4} = \frac{\sin 38}{a}$
$a = 2.6\text{ km}$.



$\sin 33 = \frac{h}{2.6}$
$h = 2.6 \sin 33$
$h = 1.4\text{ km}$

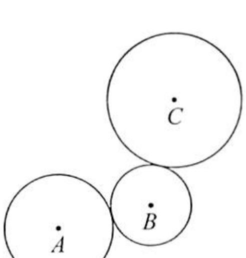
10. Two planes leave airport A in different directions. One plane lands at airport B, 630 km from airport A. The other plane lands at airport C some time later. If $\angle ABC = 110°$ and $\angle ACB = 40°$, how far did the second plane fly?

11. In a solar system, the distance from the Sun (S) to planets A and B are 85 and 61 million miles respectively. When $\angle A = 20°$, how far is it from planet A to planet B and B'?



$\frac{\sin 20}{61} = \frac{\sin B'}{85}$.
$\angle B' = 28.46°$.
$\angle SBA = 151.54°$.
$\frac{\sin 151.54}{85} = \frac{\sin 8.46}{BA}$
$BA = 26$ million miles.
$\angle B'SA = 180 - 28.46 - 20 = 131.54°$
$\frac{\sin 131.54}{AB'} = \frac{\sin 20}{61}$ $AB' = 133$ million miles.

12. Three circles with radius $A = 4$ cm, $B = 3$ cm, and $C = 5$ cm are shown. If $\angle CAB = 35°$, how far is it from the centre of circle A to the centre of circle C?



3.6 Exercise Set

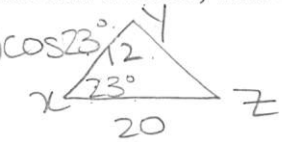
1. Fill in the blank.

a) Use the Law of Cosines when the information given for the triangle is SSS or SAS.

b) If the Law of Cosines $a^2 = b^2 + c^2 - 2bc\cos A$ is applied to a right triangle, the result is pythagorean theorem, since $\cos 90° =$ 0.

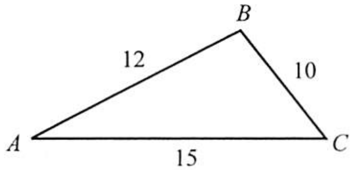
$a^2+b^2=c^2$

c) Write a version of the Law of Cosines that is needed to solve ΔXYZ, with $\angle YXZ = 23°$, $z = 12$, and $y = 20$. $x^2=12^2+20^2-2(12)(20)\cos 23°$

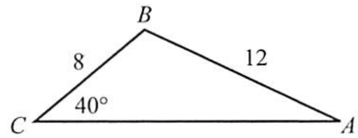


2. Determine whether the Law of Sines or the Law of Cosines would be used to begin the solution process for each triangle.

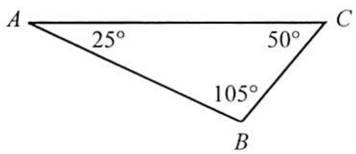
a) C



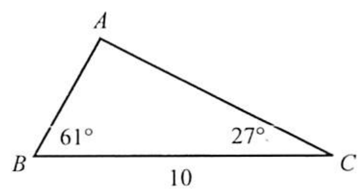
b) S



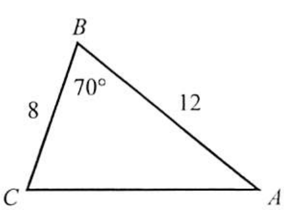
c) neither



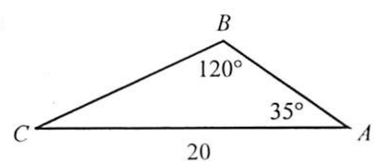
d) S



e) C



f) S



3. Solve each Law of Cosine for the unknown part. Answer to 2 decimal places.

a) $a^2 = 5^2 + 3^2 - 2 \cdot 5 \cdot 3\cos 43°$

$a=3.47$

b) $b^2 = 7^2 + 8^2 - 2 \cdot 7 \cdot 8\cos 115°$

c) $c^2 = 4^2 + 6^2 - \cancel{2 \cdot 4 \cdot 6\cos 90°}$ zero

$c=7.21$

d) $7^2 = 3^2 + 6^2 - 2 \cdot 3 \cdot 6\cos A$

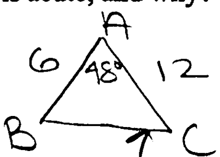
e) $5.3^2 = 2.7^2 + 4.6^2 - 2(2.7)(4.6)\cos B$

$\cos^{-1}\left(\frac{5.3^2-2.7^2-4.6^2}{-2(2.7)(4.6)}\right)$

$\angle B=89.17°$

f) $9.3^2 = 6.2^2 + 4.5^2 - 2(6.2)(4.5)\cos C$

4. In ΔABC, if $\angle A = 48°$, $b = 12$, and $c = 6$, which of the two angles $\angle B$ or $\angle C$ can be said for certain is acute, and why?

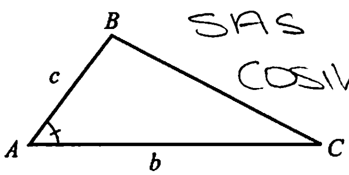


must be smaller than $\angle B$ since $6 < 12$

5. In ΔABC, if $\angle A = 95°$, $b = 5$, and $c = 9$, which of the two angles $\angle B$ or $\angle C$ can be said for certain is acute, and why?

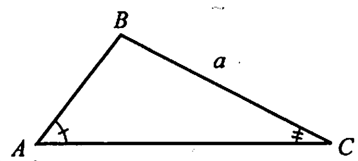
6. Given the indicated parts of ΔABC, what angle or side should be found first, and which formula should be used to find it?

a)

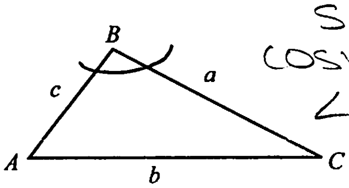


SAS
cosine Law
a
(remaining side)

b)

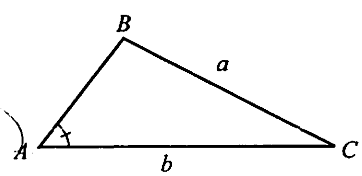


c)

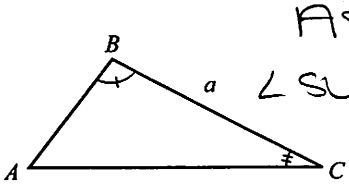


SSS
cosine Law
$\angle B$
(biggest $\angle$)

d)

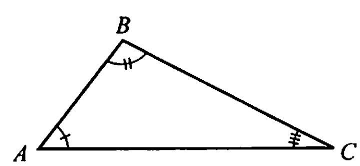


e)

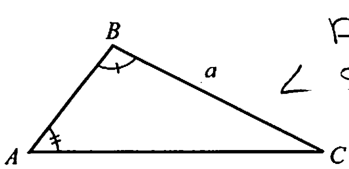


ASA
$\angle$ sum of Δ
$\angle A$

f)

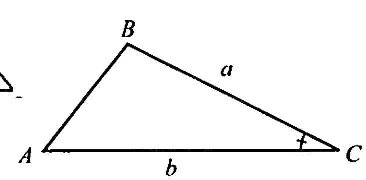


g)



AAS
$\angle$ sum of Δ
$\angle C$

h)



7. Solve the triangle. Round answers to one decimal place.

a) $\angle A = 50°,\ b = 10,\ c = 15$

$a^2 = 15^2 + 10^2 - 2(15)(10)\cos 50$

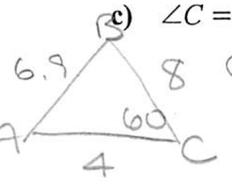
$a = 11.5$

$\frac{\sin B}{10} = \frac{\sin 50}{11.5}$ $\quad \angle B = 41.8°$

$\angle C = 180 - 50 - 41.8 = 88.2°$

b) $\angle B = 36°,\ a = 4,\ c = 10$

c) $\angle C = 60°,\ b = 4,\ a = 8$



$c^2 = 4^2 + 8^2 - 2(4)(8)\cos 60$

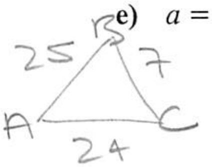
$c = 6.9$

$\frac{\sin B}{4} = \frac{\sin 60}{6.9}$ $\quad \angle B = 30.1°$

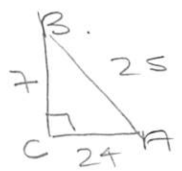
$\angle A = 89.9°$

d) $a = 2,\ b = 3,\ c = 4$

e) $a = 7,\ b = 24,\ c = 25$



$25^2 = 24^2 + 7^2 \underbrace{- 2(24)(7)\cos C}_{\text{must } = 0}$



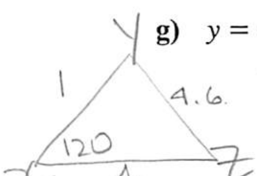
$\therefore \angle C = 90°$

$\sin A = \frac{7}{25}$ $\quad \angle A = 16.3°$

$\angle B = 73.4°$

f) $a = 9,\ b = 14,\ c = 11$

g) $y = 4,\ z = 1,\ \angle X = 120°$



$x^2 = 1^2 + 4^2 - 2(1)(4)\cos 120$

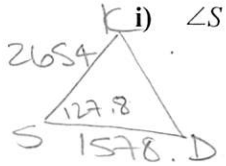
$x = 4.6$

$\frac{\sin Z}{1} = \frac{\sin 120}{4.6}$ $\quad \angle Z = 10.9°$

$\angle Y = 180 - 120 - 10.9 = 49.1°$

h) $x = 6,\ y = 7,\ z = 13$

i) $\angle S = 127.8°,\ k = 1578,\ d = 2654$



$s^2 = 1578^2 + 2654^2 - 2(1578)(2654)\cos 127.8°$

$s = 3829.8$

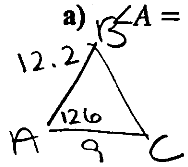
$\frac{\sin K}{1578} = \frac{\sin 127.8}{3829.8}$ $\quad \angle K = 19.0°$

$\angle D = 33.2°$

j) $s = 1504,\ q = 2365,\ r = 1953$

8. Solve ΔABC using either the Law of Sines or the Law of Cosines to begin the solution.

a) $\angle A = 126°,\ b = 9,\ c = 12.2$



$a^2 = 12.2^2 + 9^2 - 2(12.2)(9)\cos 126$

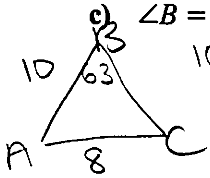
$a = 18.9$

$\frac{\sin B}{9} = \frac{\sin 126}{18.9}$ $\quad \angle B = 22.7°$

$\angle C = 31.3°$

b) $\angle A = 28°,\ \angle B = 42°,\ c = 18.2$

c) $\angle B = 63°,\ b = 8,\ c = 10$



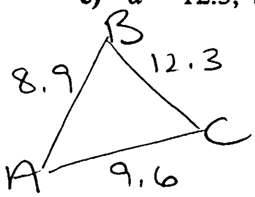
$10\sin 63 \cancel{<} 8 < 10$

$\frac{\sin 63}{8} = \frac{\sin C}{10}$

$\frac{10\sin 63}{8} > 1 \therefore \varnothing$

d) $\angle B = 41°,\ a = 11,\ c = 6$

e) $a = 12.3,\ b = 9.6,\ c = 8.9$



$12.3^2 = 8.9^2 + 9.6^2 - 2(8.9)(9.6)\cos A$

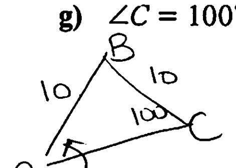
$\angle A = 83.3°$

$\frac{\sin C}{8.9} = \frac{\sin 83.3}{12.3}$ $\quad \angle C = 45.9°$

$\angle B = 180 - 83.3 - 45.9 = 50.8°$

f) $\angle C = 38°,\ b = 9,\ c = 7$

g) $\angle C = 100°,\ a = 10,\ c = 10$

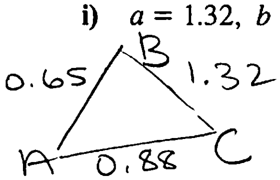


can't equal 100°

$\varnothing$

h) $\angle A = 60°,\ a = 2\sqrt{3},\ c = 4$

i) $a = 1.32,\ b = 0.88,\ c = 0.65$



$1.32^2 = 0.65^2 + 0.88^2 - 2(.65)(.88)\cos A$

$\angle A = 118.5°$

$\frac{\sin C}{0.65} = \frac{\sin 118.5}{1.32}$ $\quad \angle C = 25.6°$

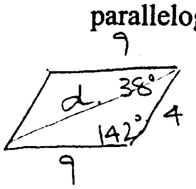
$\angle B = 180 - 118.5 - 25.6 = 35.9°$

j) $\angle A = 75°,\ b = 4 - 2\sqrt{3},\ a = \sqrt{6} - \sqrt{2}$

9. A plane flies 420 km from point A at a direction of 135° from due east and then travels 240 km at a direction of 240° from due east. How far is the plane from point A?

10. Two planes leave Victoria at 9 am. One plane travels due east at 500 km/h, while the other plane travels 640 km/h $N\,30°\,W$. How far apart are the two planes at noon?

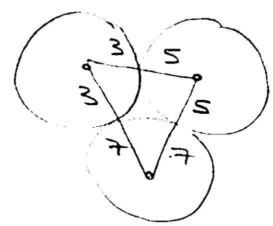
11. Two adjacent sides of a parallelogram meet at an angle of 38° and have lengths of 4 cm and 9 cm. What is the length of the larger diagonal of the parallelogram?

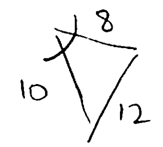


$d^2 = 9^2 + 4^2 - 2(9)(4)\cos 142$

$d = 12.4$ cm

12. Three circles of radius 3, 5, and 7 cm are tangent to each other. Find the largest angle formed by joining their centres.



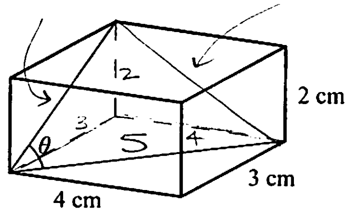


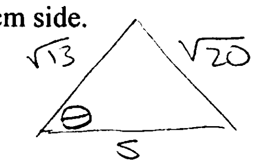
$12^2 = 10^2 + 8^2 - 2(10)(8)\cos C$

$\angle C = 82.8°$

$\sqrt{9+4} = \sqrt{13}$ $\sqrt{16+4} = \sqrt{20}$

13. The rectangular box has dimensions 4 cm × 3 cm × 2 cm. Find angle θ formed by a diagonal of the base, and a diagonal of the 2 cm × 3 cm side.

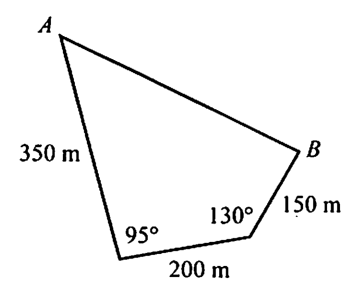




$\sqrt{20}^2 = \sqrt{13}^2 + 5^2 - 2(5)(\sqrt{13})\cos\theta$

$\theta = 60.1°$

14. An irregular plot of land has dimensions as shown. Find AB.



3.7 Chapter Review

Section 3.1

1. Determine the smallest positive coterminal angle for the given angle.

a) $-20°$ $+360$ $\underline{340°}$ b) $-100°$ ______

c) $-240°$ $+360$ $\underline{120°}$ d) $-280°$ ______

e) $400°$ -360 $\underline{40°}$ f) $500°$ ______

2. Find the reference angle for the following.

a) $73°$ $\underline{73°}$ b) $137°$ ______

c) $291°$ $360-291$ $\underline{69°}$ d) $204°$ ______

e) $-137°$ $180-137$ $\underline{43°}$ f) $-291°$ ______

g) $-204°$ $204-180$ $\underline{24°}$ h) $1000°$ ______

3. Find all angles, $0° \le \theta < 360°$, that have the given reference angle.

a) $71°$ $71°$ $71°$ $71°$ $71°$ $\underline{71°, 109°,}$ $251°, 289°$ b) $43°$ ______

Section 3.2

4. Given a point on the terminal side of angle θ. Evaluate the three trigonometric functions of θ.

a) $(3,7)$ $(3,7)$ 7 3 $\sqrt{9+49}$ $=\sqrt{58}$

$\sin\theta = \frac{7}{\sqrt{58}}$

$\cos\theta = \frac{3}{\sqrt{58}}$

$\tan\theta = \frac{7}{3}$

b) $(-1, 2\sqrt{2})$

c) $(-2\sqrt{3}, 2)$ 2 $-2\sqrt{3}$ $\sqrt{4+4(3)}$ $=4$

$\sin\theta = \frac{2}{4} = \frac{1}{2}$

$\cos\theta = -\frac{2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$ $\tan\theta = \frac{2}{-2\sqrt{3}} = -\frac{1}{\sqrt{3}}$

d) $(-\sqrt{17}, -2\sqrt{2})$

5. Given a linear equation of the terminal side of angle θ, with a restriction, find the value of the three trigonometric functions of θ.

$\sqrt{9+4}$

a) $y = \frac{2}{3}x,\ x \geq 0$ $\quad \sin\theta = \frac{2}{\sqrt{13}}$

$\sqrt{13}$ 2 $\quad \cos\theta = \frac{3}{\sqrt{13}} \quad \tan\theta = \frac{2}{3}$

3

b) $y = \frac{2}{3}x,\ x \leq 0$

c) $y = -\frac{1}{4}x,\ x \geq 0$

4 $\quad -1 \quad \sqrt{17}$

$\sin\theta = \frac{-1}{\sqrt{17}} \quad \cos\theta = \frac{4}{\sqrt{17}}$

$\tan\theta = -1/4$

d) $y = -\frac{1}{4}x,\ x \leq 0$

6. Given one of the three primary trigonometric functions, find the other two trigonometric functions of θ.

a) $\sin\theta = \frac{2}{\sqrt{5}}$, θ in quadrant I $\quad \sqrt{5}$ 2 1

$\cos\theta = \frac{1}{\sqrt{5}} \quad \tan\theta = \frac{2}{1} = 2$

b) $\tan\theta = -\frac{2}{\sqrt{21}}$, θ in quadrant II

c) $\cos\theta = -0.416$, θ in quadrant III

65.4

$\sin\theta = -0.909$

$\tan\theta = 2.184$

d) $\sin\theta = -0.421$, θ in quadrant IV

Section 3.3

7. Find all θ, $0° \leq \theta < 360°$, which satisfy each equation.

a) $\sin\theta = \frac{\sqrt{2}}{2}$ $\quad \frac{1}{\sqrt{2}}$ $\quad$ 45 $\sqrt{2}$ 1 $\quad \theta = 45°, 135°$

b) $\cos\theta = -\frac{\sqrt{3}}{2}$

c) $\tan\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$ $\quad -\sqrt{3}$ 30° 1 $\sqrt{3}$ -1 $\quad \theta = 30°, 210°$

d) $\sin\theta = 0$

e) $\cos\theta = 0.7071$ $\quad$ 45 $\quad \theta = 45°, 315°$

f) $\tan\theta = -1.732$

Section 3.4

8. Solve ΔABC by using right triangles, not by using the Law of Sines or Cosines.

a) $\angle A = 40°,\ \angle B = 60°,\ b = 8$

b) $a = 4,\ b = 5,\ c = 6$

c) $\angle C = 47°,\ a = 8,\ b = 5$

d) $\angle B = 110°,\ \angle C = 32°,\ a = 5$

Sections 3.5, 3.6

9. Solve ΔABC using the Law of Sines or Law of Cosines to begin the solution.

a) $\angle B = 104°,\ a = 17,\ c = 11$

$b^2 = 11^2 + 17^2 - 2(11)(17)\cos 104$

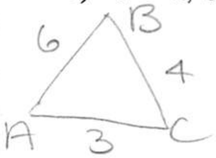
$b = \boxed{22.4}$

$\frac{\sin C}{11} = \frac{\sin 104}{22.4}$ $\angle C = \boxed{28.5°}$

$\angle A = \boxed{47.5°}$

b) $\angle A = 40°,\ \angle B = 40°,\ c = 2$

c) $a = 4,\ b = 3,\ c = 6$



$6^2 = 3^2 + 4^2 - 2(3)(4)\cos C$

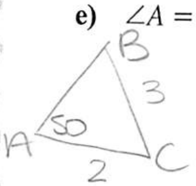
$\angle C = \boxed{117.3°}$

$\frac{\sin B}{3} = \frac{\sin 117.3}{6}$ $\angle B = \boxed{26.4°}$

$\angle A = \boxed{36.3°}$

d) $\angle A = 95°,\ \angle B = 45°,\ a = 5$

e) $\angle A = 50°,\ a = 3,\ b = 2$



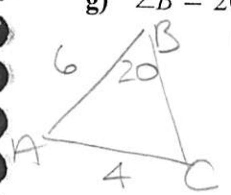
$\frac{\sin 50}{3} = \frac{\sin B}{2}$

$\angle B = 30.7°$ $\angle C = 99.3°$

$\frac{\sin 99.3}{c} = \frac{\sin 50}{3}$ $c = 3.9$

f) $\angle A = 60°,\ a = 4,\ b = 5$

g) $\angle B = 20°,\ b = 4,\ c = 6$



$6\sin 20 < 4 < 6$ $\therefore 2\Delta$'s.

$\frac{\sin 20}{4} = \frac{\sin C}{6}$ $\angle C = 30.9°$ or $149.1°$

$\angle A = 129.1°$ or $10.9°$

$\frac{\sin 20}{4} = \frac{\sin 129.1}{a}$ $a = 9.1$ or 2.2

$\frac{\sin 20}{4} = \frac{\sin 10.9}{a}$

h) $\angle C = 60°,\ a = 2\sqrt{6},\ c = 3\sqrt{2}$

10. In ΔABC, $b < a < c$. What does this imply about angles A, B, and C?

smallest ∠. largest ∠

11. Given ΔABC, with angle θ between sides b and c, find θ if $a^2 = b^2 + c^2 + bc$.